COMPLEX ADAPTIVE SYSTEMS LABORATORY

From Graphons to Graphlets: A Graph-Limit Lens on Sparse Neural Connectivity and Training

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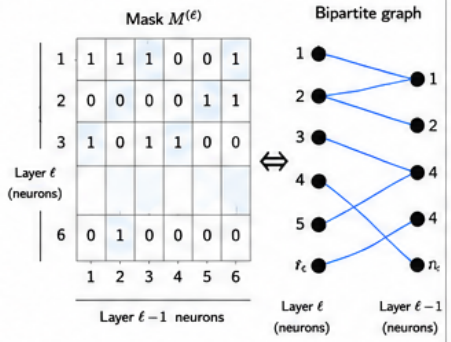
1 Pruning masks are graphs

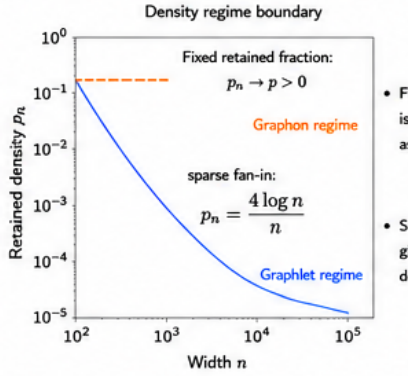
A layerwise pruning mask is naturally a bipartite graph.

$$M^{(\ell)} \in \{0,1\}^{n_\ell \times n_{\ell-1}}$$

As width grows, the right limit object depends on the sparsity regime.

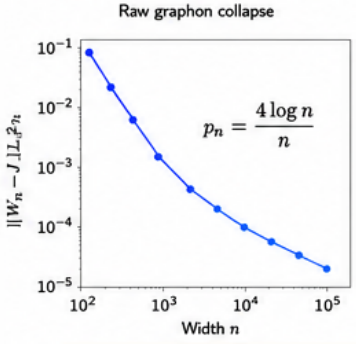


2 Two sparsity regimes



- Fixed 10% pruning is still asymptotically dense.
- Sublinear fan-in gives vanishing density.

3 Why raw graphons fail in the sparse fan-in regime



Half-graph limit intuition

As $n \to \infty$,

$$W(x,y) = \begin{cases} 1 & \text{if } x+y \le \frac{1}{2} \\ 1 & \text{if } x+y > \frac{1}{2} \\ 0 & \text{otherwise.} \end{cases}$$

If $p_n \to 0$, then $W_{\pi,n} \to 0$ in L^1 and in cut norm.

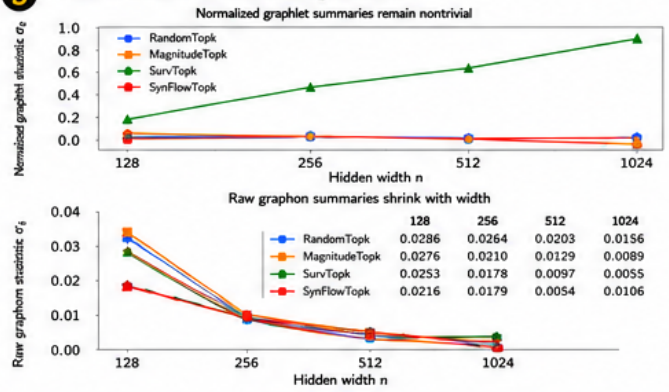
4 Graphlets rescue sparse structure

$$G_n = (\mu_{L,n}, \mu_{R,n}, T_n, P_n, \Phi_n)$$

$$T_n = D_L^{-1/2} M_n D_R^{-1/2}$$
$$P_n = D_L^{-1} M_n$$

- Degree measures, normalized incidence, routing, and volume survive density collapse.
- Graphlets are the density-aware limit object for sparse neural masks.

5 Neural masks remain distinguishable



	128	256	512	1024
RandomTopk	0.0286	0.0264	0.0203	0.0156
MagnitudeTopk	0.0276	0.0210	0.0129	0.0089
SurvTopk	0.0253	0.0178	0.0097	0.0055
SynFlowTopk	0.0216	0.0179	0.0054	0.0106

Raw summaries mostly track density collapse; normalized graphlet summaries remain method-dependent.

6 Toward Graphlet NTK

$$z_i(x) = \sum_j \frac{M_{ij}}{\sqrt{d_j}} W_j h_j(x)$$

- With fan-in normalization, covariance propagation is governed by the routing operator $P = D^{-1}M$.
- Graphon NTK is appropriate when $p_n \to p > 0$.
- Graphlet-based routing operators are the natural sparse analogue when $p_n \to 0$.
- Graphlet NTK is ongoing work.

Raw summaries mostly track density collapse; normalized graphlet summaries remain method-dependent.

7 Main takeaway

Dense / fixed-fraction regime	$p_n \to p > 0$	$\Rightarrow$	graphons / Graphon NTK
Sparse fan-in regime	$p_n \to 0$	$\Rightarrow$	graphlets / sparse routing operators

Graphons before density collapse. Graphlets after density collapse.