Emergence of Giant Component in Percolated Artificial Neural Networks

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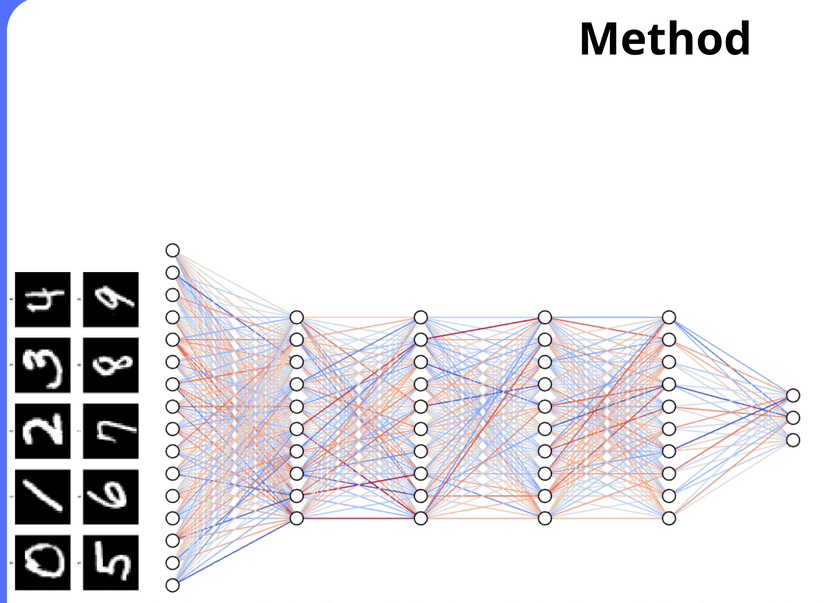
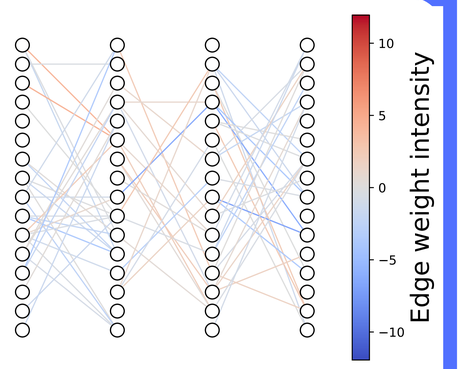
Context

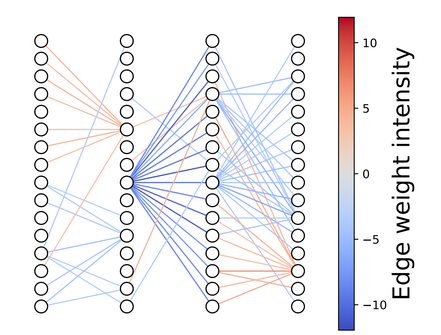
- Motivated by pruning techniques, we study the robustness of Multi-Layer Perceptron (MLPs) to edge removal by tracking how their largest connected component evolves.
- Recent work has shown that heavy-tailed structure emerges in such networks; we show that this structure shapes their robustness.
- Finally, we show that a lognormal distribution provides a good fit to the absolute strength of neurons.

Method

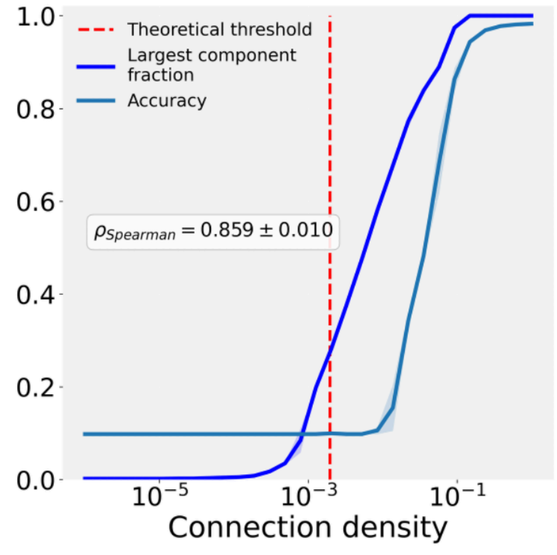


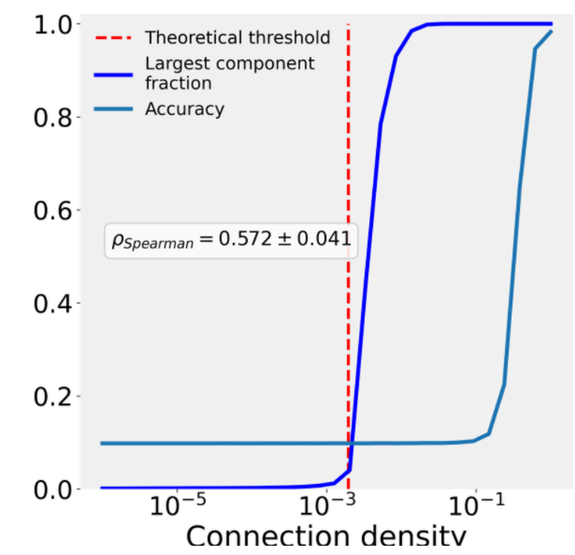


Bernoulli percolation



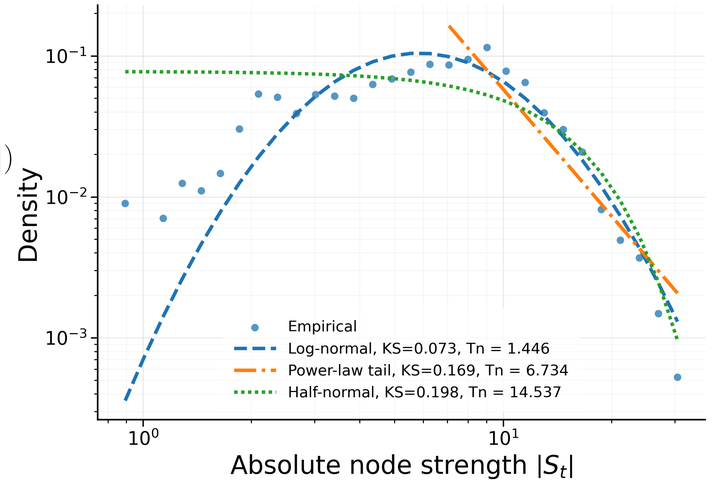
Descending percolation





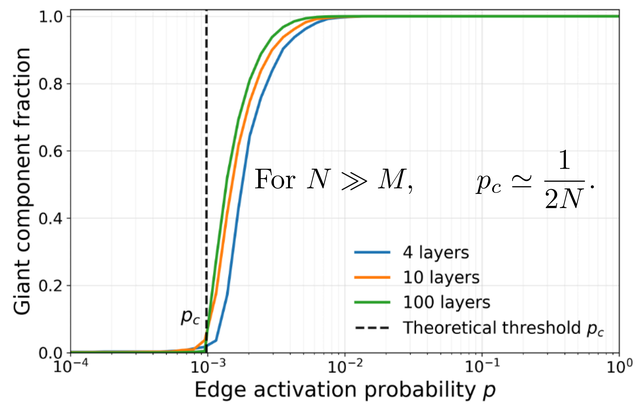
$$S_i = \sum_j w_{ij} \quad \longrightarrow \quad |S_i| \quad \longrightarrow \quad P(|S|)$$

$$P(|S|) = \frac{1}{|S|\,\sigma\sqrt{2\pi}} \exp\left[-\frac{(\ln|S| - \mu)^2}{2\sigma^2}\right]$$



Results

We performed Bernoulli percolation on MLP-like networks and found that the percolation threshold is well approximated by a value independent of the number of layers.



Conclusion

- We derived an approximation of the Bernoulli percolation threshold for feed-forward networks.
- Under descending percolation, the emergence of the giant component is strongly correlated with the emergence of network performance.
- Neural network robustness under pruning is inherited from underlying heavy-tailed structures. Here we observed that the absolute strength distribution is well fitted by a lognormal distribution.

References

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