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From Graphons to Graphlets: A Graph-Limit Lens on Sparse Neural Connectivity and Training

Siddhi Kanta Mishra and Ivan Garibay
University of Central Florida
Complex Adaptive Systems Laboratory

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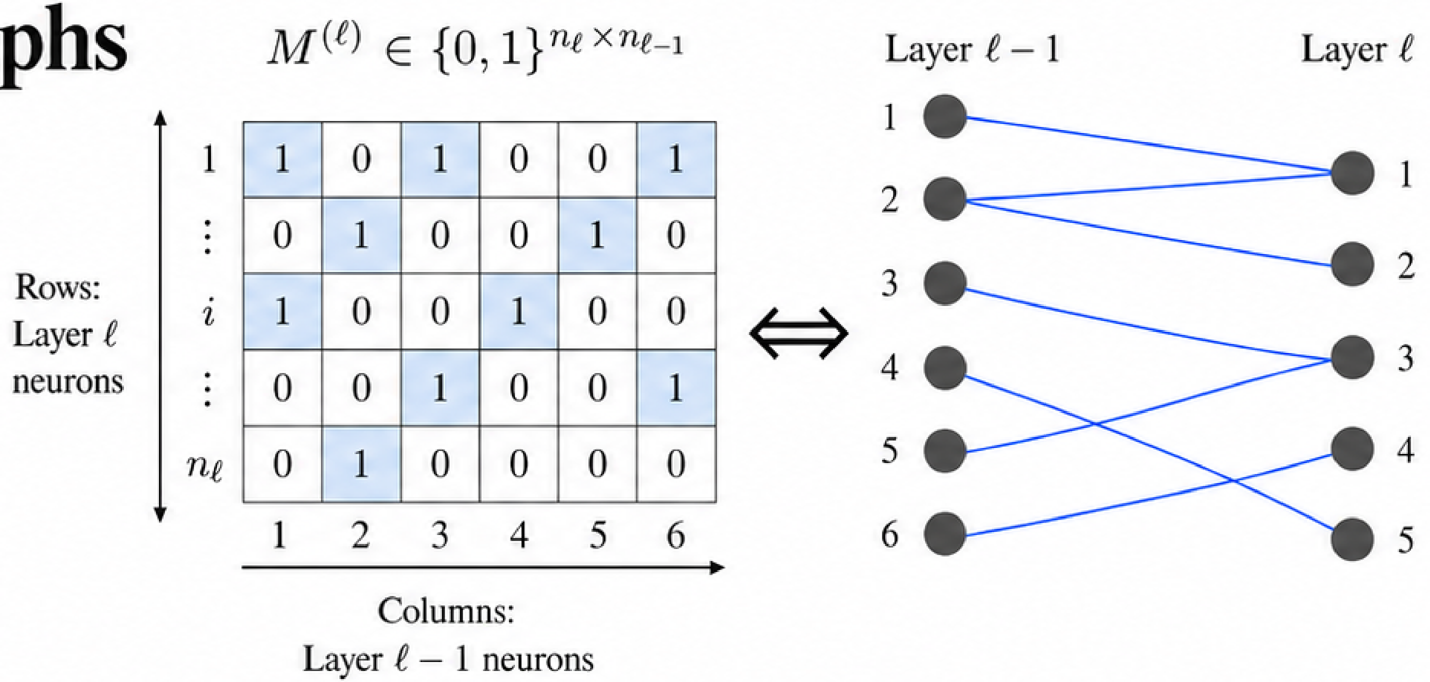
1. Pruning masks are graphs

$$M^{(\ell)} \in \{0,1\}^{n_\ell \times n_{\ell-1}}$$

- A layerwise pruning mask is a bipartite graph.
- Source neurons: layer $\ell - 1$.
- Target neurons: layer ℓ.
- Retained weights are edges.



> *When we prune a neural network, we are not only removing parameters. We are creating a network.*

2. As width grows, what does a sparse neural mask converge to?

$$M_n \longrightarrow ?$$

Fixed retained fraction

$$p_n \to p > 0$$

Dense / fixed-fraction pruning

Graphon-friendly regime

Sparse fan-in

$$k_n = o(n)$$

$$p_n = \frac{k_n}{n} \to 0$$

Sublinear fan-in

Vanishing-density regime

- Fixed retained fraction: $p_n \to p > 0$.
- Sparse fan-in: $k_n = o(n)$, so $p_n = k_n/n \to 0$.
- These are different graph-limit regimes.

> *Existing Graphon NTK work assumes pruning masks converge to graphons at fixed sparsity level $p > 0$.*

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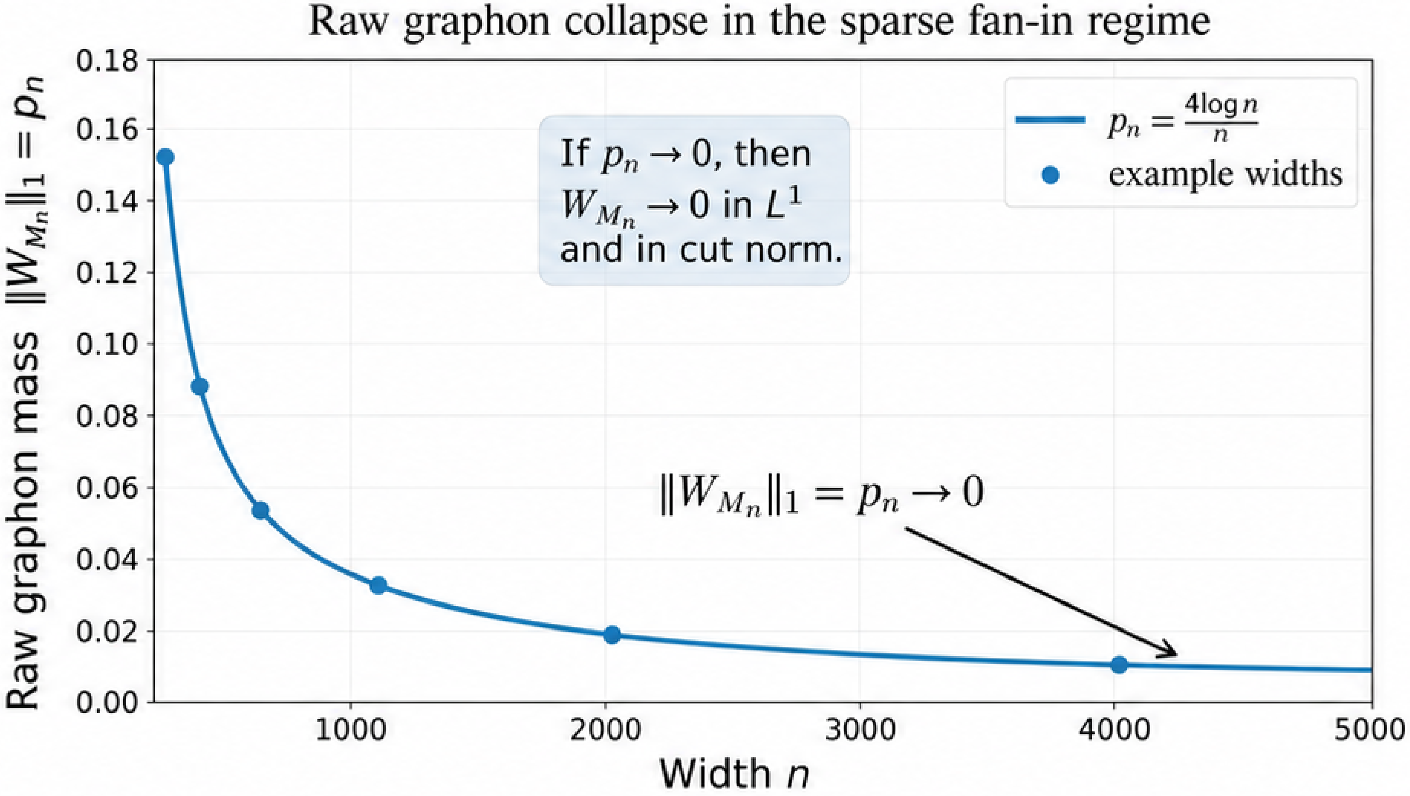
3. Raw graphons collapse when density vanishes

$$\|W_{M_n}\|_1 = \frac{1}{n_L n_R}\sum_{i,j} M_{ij} = p_n.$$

$$p_n \to 0 \implies W_{M_n} \to 0 \text{ in } L^1 \text{ and cut norm.}$$

- This is not a failure of graphons in dense regimes.
- It is a density boundary.
- Raw adjacency mass disappears before normalized structure disappears.



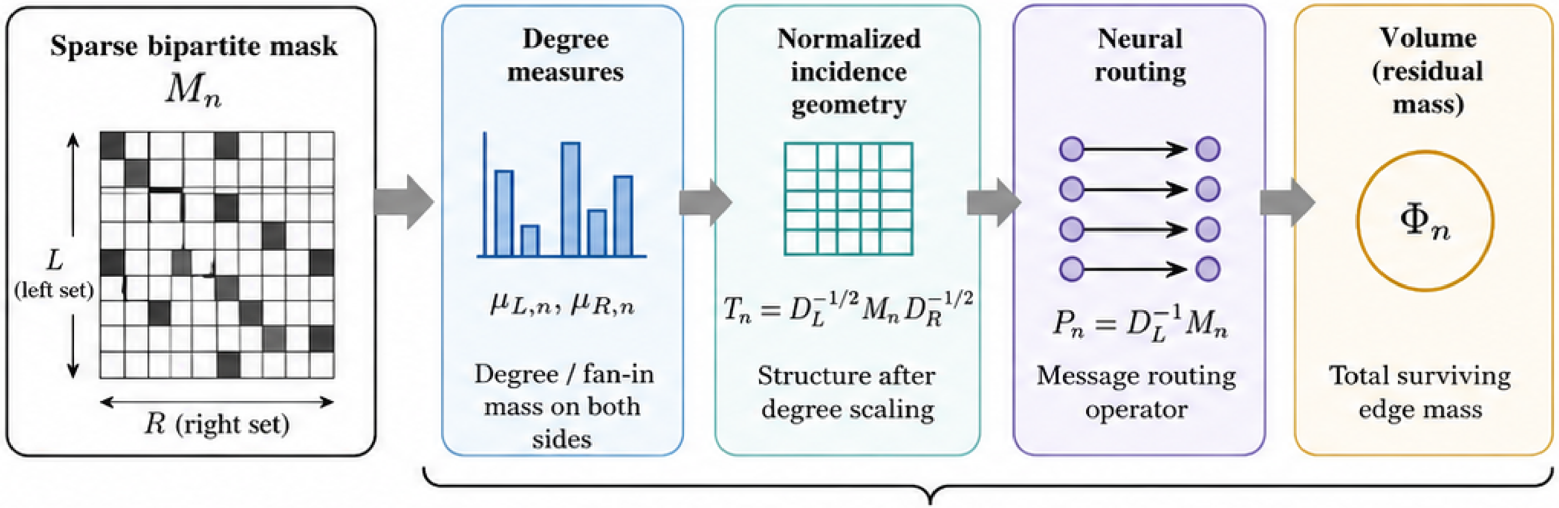
> *The theorem is almost embarrassingly simple, but it tells us exactly where the graphon coordinate loses information.*

4. Neural mask graphlets

$$G_n = (\mu_{L,n}, \mu_{R,n}, T_n, P_n, \Phi_n).$$

$$T_n = D_L^{-1/2} M_n D_R^{-1/2}, \qquad P_n = D_L^{-1} M_n.$$

- Φ_n: volume / total edge mass.
- μ_L, μ_R: degree mass on both sides.
- T_n: normalized incidence geometry.
- P_n: neural routing operator.



Graphlet decomposition: separates global sparsity, degree/fan-in mass, normalized geometry, routing, and residual structure beyond degree sequence.

> *The graphlet does not just ask how many edges survive. It asks how the surviving edge mass routes signals.*

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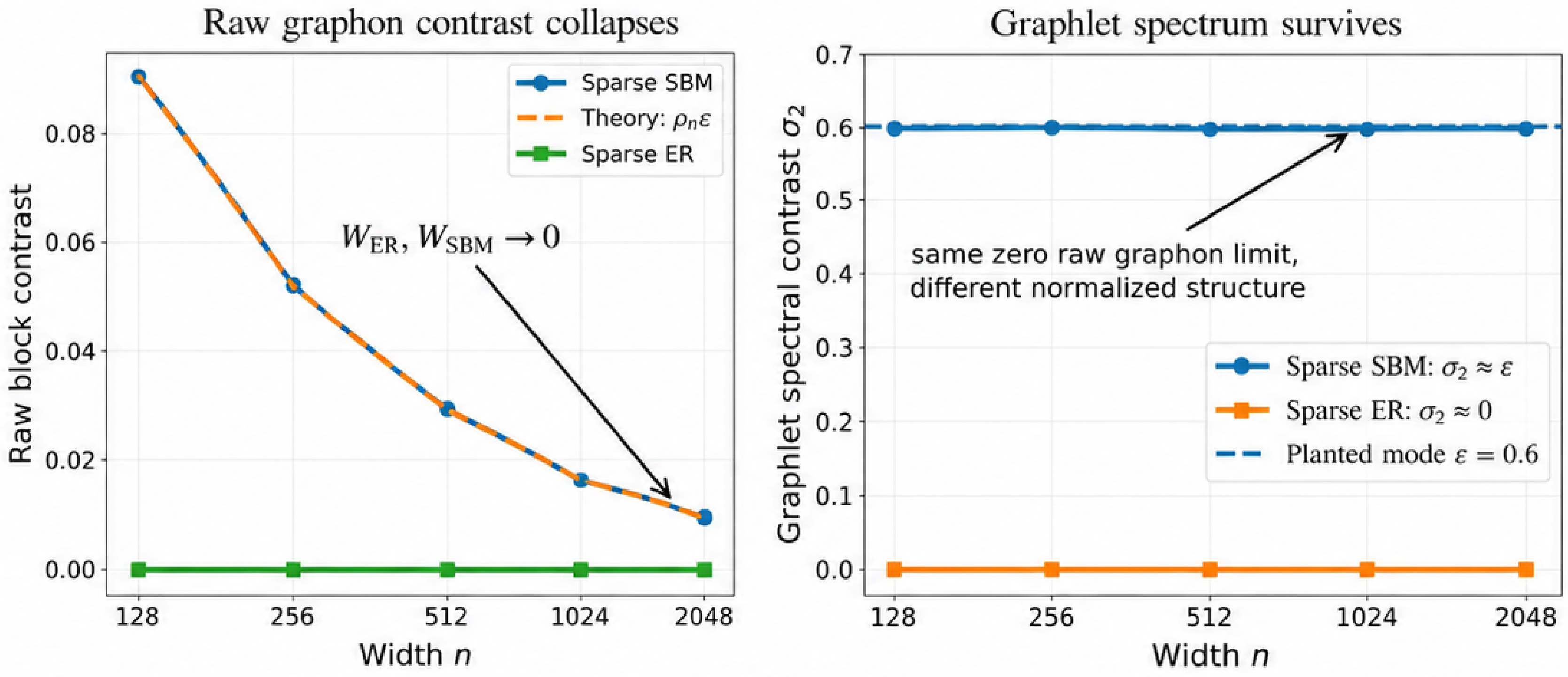
5. Same zero graphon limit, different sparse structure

sparse ER *vs.* sparse SBM

- Raw graphon contrast goes to 0 for both sparse ER and sparse SBM.
- Graphlet spectrum separates them.
- Raw graphons see both as zero.
- Graphlets retain the planted sparse mode.
- Normalization cancels the vanishing density factor.

$$\sigma_2(\mathrm{ER}) \approx 0, \quad \sigma_2(\mathrm{SBM}) \approx \varepsilon.$$

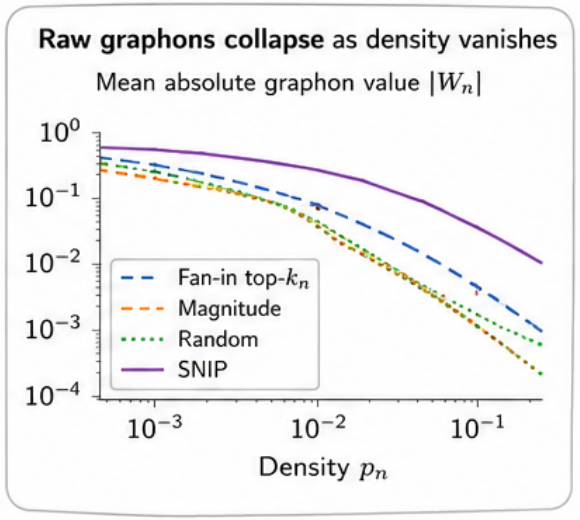
Sparse ER vs sparse SBM: graphon collapse, graphlet survival



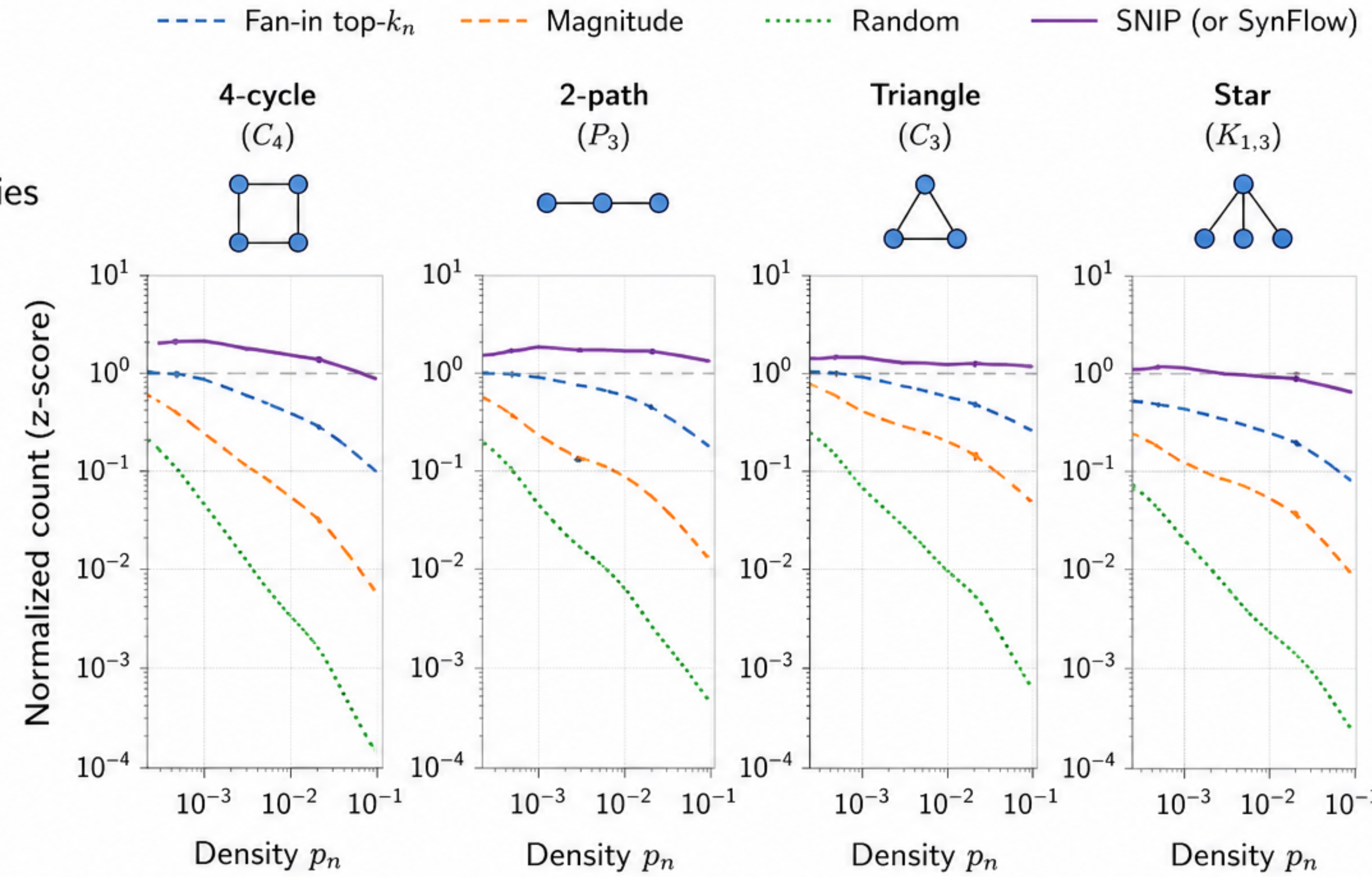
> *Raw graphons erase vanishing-density structure; graphlets retain the planted sparse mode.*

Neural masks remain distinguishable after normalization

- Fan-in top-k_n pruning gives $p_n = k_n/n \to 0$.
- Raw graphon summaries mostly shrink with density.
- Normalized graphlet summaries remain method-dependent.
- SNIP-like masks show stronger graphlet modes.



B. Normalized graphlet summaries (nontrivial as $p_n \to 0$)



Caution: These are finite-width diagnostics, not yet a full theorem for arbitrary learned pruning.

Toward sparse neural training dynamics

- Fan-in normalization makes **routing** central.
- Covariance propagation sees P, not raw M.
- This suggests a **Graphlet NTK** for $p_n \to 0$.

$$z_i(x) = \sum_j \frac{M_{ij}}{\sqrt{d_i}} \; W_{ij} \; h_j(x).$$

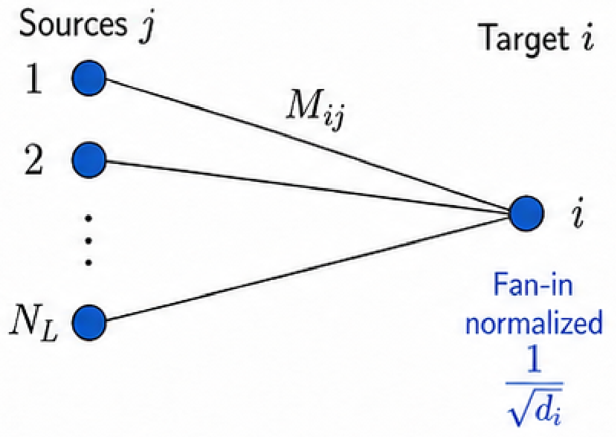
Covariance intuition:

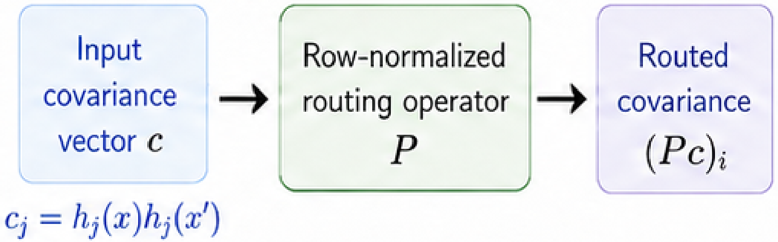
$$\mathrm{Cov}(z_i(x), z_i(x')) \sim \sum_j \frac{M_{ij}}{d_i} h_j(x) h_j(x') = (Pc)_i,$$

where $c_j = h_j(x) h_j(x')$.

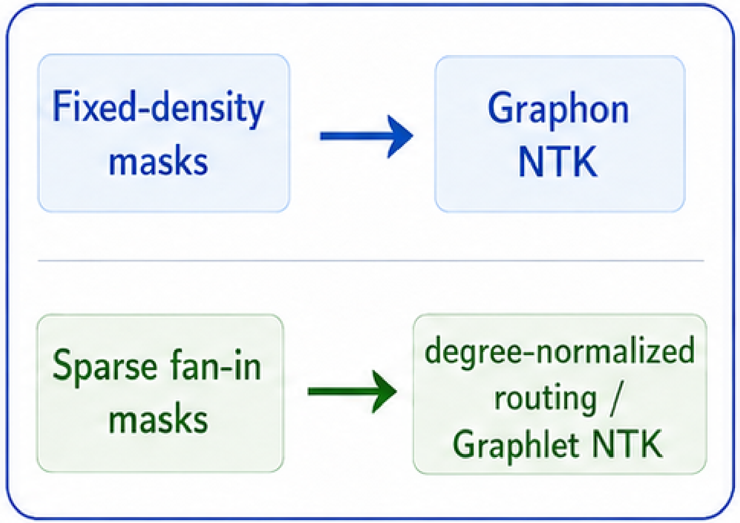
Therefore: $P = D_L^{-1} M$.

Fan-in normalized operator view





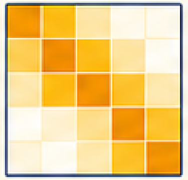
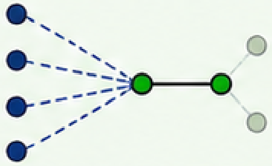
From graphon to sparse routing



Graphon NTK is natural for graphon-structured, fixed-density masks; sparse fan-in points instead to degree-normalized routing.

⚠ **Part of ongoing work.**

Graphons before density collapse. Graphlets after density collapse.

Regime	Condition $(n \to \infty)$		Interpretation
 Dense / fixed-fraction regime	$p_n \to p > 0$	$\Rightarrow$	graphons / Graphon NTK
 Sparse fan-in regime	$p_n \to 0$	$\Rightarrow$	graphlets / sparse routing operators

- Pruning masks are networks.
- Dense-style graphons are appropriate before density collapse.
- Sparse fan-in regimes require degree-aware graphlet coordinates.
- Training dynamics should be studied through normalized routing.

> *"Graphons tell us what survives before density collapse. Graphlets tell us what survives after density collapse."*

Thank You

Questions?

Key references

- Lovász, L. and Szegedy, B. (2006). Limits of Dense Graph Sequences.
- Lovász, L. (2012). Large Networks and Graph Limits.
- Chung, F. (2012). Graphlets: A Spectral Perspective for Graph Limits.
- Pham, H., Ta, T.-A., Jacobs, T., Burkholz, R., and Tran-Thanh, L. (2025).
  The Graphon Limit Hypothesis: Understanding Neural Network Pruning via Infinite Width Analysis.
- Pham, H., Ta, T.-A., and Tran-Thanh, L. (2026). Pruning at Initialisation through the Lens of Graphon Limit: Convergence, Expressivity, and Generalisation.